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FACTORIZATION METHOD
IN THE SPONTANEOUSLY BROKEN GAUGE THEORIES

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A b s t r a c t

We consider the factorization method in the spontaneously broken gauge theories such as the electroweak theory.

Using the off-shell W -boson density matrix formalism we demonstrate that the factorization conditions are completely under control.

The main point of this paper is the presence of the "interference" or "crossed" terms in WW -scattering process which exhibits itself in the dependence on the relative azimuthal angle between two scattering planes.

To illustrate our general consideration we use well-known example — a single Higgs boson production. The origine of the quantitative failure of the WW -effective method, as originally proposed, is given and it is shown how to use it correctly.

I. Introduction

In order to search for a new physics at future e^+e^- -colliders or at LHC we must well understand the fusion mechanism especially via vector W, Z bosons.

Our paper is devoted to the applicability of the effective W -boson approximation (EWA)-method. This approximate method was proposed for the calculation of $WW(ZZ)$ -fusion processes, for which there were no hopes to find exact analytical answers in general case.

Originally [1, 2] only longitudinally polarized W -boson beams were taken into account by EWA. It was found that the result of calculation of the cross-section for Higgs-boson production via WW -fusion by EWA (in the region of W -boson energy $E_W \gtrsim m_H > m_W$) is much larger than the exact total cross-section [3]. The contribution of the transversely polarized W -beams to this process only increases the discrepancy between this classical term and the exact total cross-section.

A lot of efforts were spent to resolve this discrepancy between exact computer calculations and the EWA-method. But the results were not very succesful – all the corrections were found to be small and the EWA-method was announced to be correct only for heavy Higgs-boson ($M_H \gg m_W$) and only in high energy limit $E_W \gg M_H$ [4, 5].

In this paper we suggest a resolution of this problem based on the advanced version of EWA-method developed in our paper [6]. Actually this method is well known in gamma-gamma physics [7].

The main point is that the beams of virtual W -boson are described not only by probabilities to find W -boson with longitudinal and transversal polarization but by density matrix (as it should be in quantum mechanics). We consider the nondiagonal terms in the virtual W -boson density matrix, which turn out to be of the same order of magnitude as the diagonal ones (corresponding to the longitudinal and transversal polarizations). If e^+e^- (or $q\bar{q}$) lab. frame and $W^+W^-(ZZ)$ s.m.f. are collinear the contribution from these nondiagonal elements of density matrix to the total cross section is equal to zero. This is the reason why nondiagonal terms were not considered preveously. But e^+e^- (or $q\bar{q}$) lab. frame and $W^+W^-(ZZ)$ c.m.f. are not exactly collinear and the integration over azimuthal angle in the lab. frame does not cancel the contribution from non-diagonal density matrix terms. This means that two W boson beams can not be considered as an independent, i.e. factorization property is violated in this kinematical region. We have found that these interference terms, which having a negative sing and a large absolute value, explain the discrepancy between EWA and exact computer calculation. To restore factorization (and EWA method) we have to confine kinematical region to smaller scattering angles. We demonstrate that the EWA-method correctly applied yields appropriate lower bounds for the total cross-section for any energy of colliding particles and any (small or large) mass of Higgs-boson. The contribution of each element into the cross-section depends on the dynamics of the $WW \rightarrow "X"$ subprocess (where " X " can be a Higgs boson(s), a W, Z – boson pair or a lepton pair). For example for the $WW \rightarrow H$ process (described as an example in this article) the contribution of longitudinal and spin-flip interference terms are the most important ones and the transversely polarized term's contribution is negligible (contrary to [4] P.Johnson et al.).

The paper is organized in the following way: the helicity density matrix for virtual W and Z bosons are defined in Section II. To illustrate our general consideration we use simple and well-known example – a single Higgs-boson production (see Section III). In Section IV we have discussed the exact numerical calculations of the contribution of each term in the

helicity decomposition into the total cross-section. In Section V we define the kinematical region for the application of the effective W -boson method. The results are discussed in the Conclusions.

Some results presented here, have been already published in our recent preprint [8]. In [6] we have used this approach for two Higgs boson production via WW -fusion.

II. W -boson density matrix

To describe the polarization properties of the virtual W -boson we introduce the basis of virtual W -boson helicity states. It could be done in the center of mass system (c.m.s.) of W^+ and W^-

$$q_1 = (w_1, 0, 0, q) ; \quad q_2 = (w_2, 0, 0, -q) .$$

Consider the standard set of orthonormal four-vectors orthogonal to the momentum q_1 :

$$\begin{aligned} e_\mu^{(1)}(+1) &= \frac{1}{\sqrt{2}}(0, -1, -i, 0) , \\ e_\mu^{(1)}(0) &= \frac{1}{\sqrt{-q_1^2}}(q, 0, 0, w_1) \stackrel{def}{=} -iQ_1 , \\ e_\mu^{(1)}(-1) &= \frac{1}{\sqrt{2}}(0, 1, -i, 0) \end{aligned} \quad (1)$$

and the analogous set for the second W -boson

$$\begin{aligned} e_\mu^{(2)}(\pm 1) &= e_\mu^{(1)}(\mp 1) , \\ e_\mu^{(2)}(0) &= \frac{i}{\sqrt{-q_2^2}}(-q, 0, 0, w) \stackrel{def}{=} iQ_2 , \\ e_\mu^{(2)} \cdot q_{2\mu} &= 0 . \end{aligned} \quad (2)$$

These four-vectors obviously represent the ± 1 and 0 helicity states of virtual W -bosons in their c.m.s. and form a complete orthonormal basis for the subspace orthogonal to $q_{1\mu}$ and $q_{2\mu}$ respectively:

$$e_\mu^{(1)\star}(a)e_\mu^{(1)}(b) = (-1)^a \delta_{a,b} , \quad a, b = \pm 1, 0 , \quad (3)$$

$$\sum_a e_\mu^{(1)\star}(a)e_\nu^{(1)}(a) = g_{\mu\nu} - \frac{q_{1\mu}q_{1\nu}}{q_1^2} . \quad (4)$$

Any vector or matrix orthogonal to $q_{1\mu}$ could be expanded over this basis. Further we shall neglect the masses of light quarks and leptons. In this approximation

$$\rho_{\mu\nu}^{(1)} q_{1\mu} = \rho_{\mu\nu}^{(2)} q_{2\mu} = 0 \quad (5)$$

and as a result

$$\rho_{\mu\nu}^{(1)} = \sum_{a,b} e_\mu^{(1)\star}(a)e_\nu^{(1)}(b)\rho_{a,b}^{(1)} , \quad a = \pm 1, 0 , \quad (6)$$

$$\rho_{ab}^{(1)} = (-1)^{a+b} e_\mu^{(1)}(a)[e_\nu^{(1)}(b)]^\star \rho_{\mu,\nu}^{(1)} . \quad (7)$$

Here ρ_{ab} is the density matrix in the helicity representation:

$$\rho_{ab}^{(i)} = \begin{Bmatrix} \rho_{++}^{(i)} & \rho_{+0}^{(i)} & \rho_{+-}^{(i)} \\ \rho_{0+}^{(i)} & \rho_{00}^{(i)} & \rho_{0-}^{(i)} \\ \rho_{-+}^{(i)} & \rho_{-0}^{(i)} & \rho_{--}^{(i)} \end{Bmatrix} , \quad i = 1, 2 . \quad (8)$$

Detailed expressions for Lorentz invariant density matrix elements $\rho_{ab}^{(i)}$ are presented in the Appendix I and II. This density matrix is nondiagonal. It means that the system is strongly polarized and quantum mechanical interference effects are large. The diagonal components $\rho_{++}^{(i)}$, $\rho_{--}^{(i)}$ and $\rho_{00}^{(i)}$ represent the fraction of longitudinal and transverse W -bosons inside the fermion (electron) (unnormalized classical probabilities). The nondiagonal components $\rho_{+0}^{(i)}$, $\rho_{0-}^{(i)}$ and $\rho_{+-}^{(i)}$ correspond to the spin-flip transitions of W -boson and depend on the azimuthal angle $\tilde{\varphi}_1$ in the W^+W^- c.m.s. For the relative azimuthal angle between the scattering planes of the colliding particles $\Delta\tilde{\varphi} = \tilde{\varphi}_1 - \tilde{\varphi}_2$, we have:

$$\cos \Delta\tilde{\varphi} = -\frac{(p_{1\perp}p_{2\perp})}{\sqrt{p_{1\perp}^2 \cdot p_{2\perp}^2}}, \quad (9)$$

where $p_{1\perp}$ ($p_{2\perp}$) is the perpendicular component of the electron momentum.¹

W -bosons are produced by the left fermion component only, that is why the W -boson density matrix elements are different from the photon density matrix ones [7], as opposed to photons, which produced by the left and right fermions with the same weight, and to Z -bosons ones (because Z -boson is produced by left and right fermion components, but with the different weights). These tiny effects are interesting to study at polarized e^+e^- -beams. We shall discuss them in the next publication.

III. Exact calculation of Higgs-boson production

The process of Higgs-boson production via W -boson fusion in e^+e^- (or $q\bar{q}$) collisions is shown in (Fig. 1). Its cross-section can be written in the form:

$$d\sigma = \left(\frac{\pi\alpha_w}{2}\right)^2 \cdot \frac{1}{4\sqrt{(p_1p_2)^2 - p_{1\perp}^2p_{2\perp}^2}} \frac{q_1^2}{(q_1^2 - m_w^2)^2} \frac{q_2^2}{(q_2^2 - m_w^2)^2}. \quad (10)$$

$$\left[4\rho_{\mu\nu}^{(1)}\rho_{\alpha\beta}^{(2)}\right] M_{\mu\alpha}^* M_{\nu\beta} (2\pi)^4 \delta^4(q_1 + q_2 - k) \frac{d^3\vec{k}}{(2\pi)^3 2E_k} \frac{d^3\vec{p}'_1}{(2\pi)^3 2E'_1} \cdot \frac{d^3\vec{p}'_2}{(2\pi)^3 2E'_2},$$

where $\alpha_w = \frac{\alpha}{\sin^2\theta_w}$ (α – is the fine structure constant and θ_w is a Weinberg angle); $M_{\mu\alpha}$ – denotes the amplitude for Higgs-boson production in the virtual W -boson collision, in the Standard model:

$$M_{\mu\alpha} = g m_w \cdot g_{\mu\alpha}. \quad (11)$$

Note, that the tensor structure $g_{\mu\alpha}$ of this vertex is fixed by the unitarity and is thus the same in all models, except for some scaling coefficients [9]. The tensors $\rho_{\mu\nu}^{(1)}$ and $\rho_{\alpha\beta}^{(2)}$ are defined by the currents of the colliding particles (electrons):

$$\begin{aligned} \rho_{\mu\nu}^{(1)} &= -\frac{1}{2q_1^2} \text{Tr}[(\hat{p}_1 + m_e)\gamma_\mu(\hat{p}'_1 + m_e)\gamma_\nu(1 + \gamma_5)] = \\ &= -\left(g_{\mu\nu} - \frac{q_{1\mu}q_{1\nu}}{q_1^2}\right) - \frac{(2p_1 - q_1)_\mu(2p_1 - q_1)_\nu}{q_1^2} + 2i\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}q_{1\beta} + g_{\mu\nu}\frac{m_e^2}{q_1^2} \end{aligned} \quad (12)$$

¹Note, that the W -density matrix is defined only by the fermion- W -fermion vertex and have no dependence on the Higgs-boson models.

and represent spin density matrix for effective W -bosons.

If we substitute (11, 12) in formula (10) we shall obtain the known [10, 11, 13] expression for the matrix element squared:

$$\left[\rho_{\mu\nu}^{(1)} \rho_{\alpha\beta}^{(2)} \right] M_{\mu\alpha}^* M_{\nu\beta} = \frac{16}{q_1^2 q_2^2} (p_1 p_2) (p'_1 p'_2) \quad (13)$$

where p_1, p_2 (p'_1, p'_2) are the four-momenta of the incident (final) electrons (quarks) (Fig.1), we have verified that the exact formula (10) with (13) gives numerically the same result [12] as the previous calculations [2, 10, 11]. (One integration with δ -function has been done analytically, see Appendix III.) Below, we shall discuss the contribution of the different W -boson polarization states in the total cross-section.

IV. Exact calculation of the input of different W -boson polarization states into the total cross-section

Using the helicity representation for $\rho_{\mu\nu}^{(1)}$ and $\rho_{\mu\nu}^{(2)}$ (6) - (8) one can rewrite the total cross-section (10) in the following way [6]

$$\begin{aligned} d\sigma = & \frac{1}{4(p_1 p_2)} \left(\frac{\pi \alpha_w}{2} \right)^2 \frac{q_1^2}{(q_1^2 - m_w^2)^2} \frac{q_2^2}{(q_2^2 - m_w^2)^2} 16\sqrt{X} \cdot \left\{ \rho_{00}^{(1)} \rho_{00}^{(2)} \hat{\sigma}_{LL} + \right. \\ & + \left(\rho_{++}^{(1)} \rho_{++}^{(2)} + \rho_{--}^{(1)} \rho_{--}^{(2)} \right) \hat{\sigma}_{TT} + 2 \left(\rho_{+0}^{(1)} \rho_{+0}^{(2)} + h.c. \right) \mathcal{J}_{LT} + \\ & \left. + \left(\rho_{+-}^{(1)} \rho_{+-}^{(2)} + h.c. \right) \mathcal{J}_{TT} \right\} \frac{1}{28\pi^4} d\zeta d\eta dq_1^2 dq_2^2 \frac{d(\Delta\phi)}{2\pi}, \end{aligned} \quad (14)$$

where $\zeta = \frac{2w_1}{\sqrt{S}}$; $\eta = \frac{2w_2}{\sqrt{S}}$; $\Delta\phi = \phi_1 - \phi_2$ - is the relative azimuthal angle in e^+e^- - c.m.s.

$$\cos \Delta\phi = - \frac{(q_{1\perp} \cdot q_{2\perp})}{\sqrt{q_{1\perp}^2 \cdot q_{2\perp}^2}}, \quad (15)$$

where $q_{1\perp}$ ($q_{2\perp}$) - is the perpendicular momentum of W^+ (W^-) in e^+e^- - c.m.s., \sqrt{X} - is the flux of the virtual W -bosons:

$$X = (q_1 q_2)^2 - q_1^2 q_2^2, \quad (16)$$

$\hat{\sigma}_{LL}$, and $\hat{\sigma}_{TT}$ represent the "cross-section" of the corresponding virtual W -boson components [6]:

$$\hat{\sigma}_{LL} = \frac{\pi^2 \alpha_w}{2\sqrt{X}} m_w^2 \frac{(q_1 q_2)^2}{q_1^2 q_2^2} \cdot 4\delta(\hat{s} - M_H^2), \quad (17)$$

$$\hat{\sigma}_{TT} = \frac{\pi^2 \alpha_w}{2\sqrt{X}} m_w^2 \cdot 4\delta(\hat{s} - M_H^2) \quad (18)$$

and \mathcal{J}_{LT} , \mathcal{J}_{TT} - represent the interference terms [6]:

$$\mathcal{J}_{LT} = \frac{\pi^2 \alpha_w}{2\sqrt{X}} m_w^2 \frac{(q_1 q_2)^2}{\sqrt{q_1^2 q_2^2}} \cdot 4\delta(\hat{s} - M_H^2), \quad (19)$$

$$\mathcal{J}_{TT} = \hat{\sigma}_{TT}, \quad (20)$$

we have verified that exact calculation of the total cross-section (14) with the substitution (15-20) gives exactly the same result as formula (10) [12] (see Appendix IV).

Now, being sure of the normalization, we can look at the contribution of each (17-20) term, multiplied by the corresponding density matrix element in (14), into the total cross-section. The numerical calculation of the contribution of the longitudinally and transversely polarized W -bosons into the total cross-section (14) $(\sigma_{LL} + \sigma_{TT})^{exact} / \sigma_{total}^{exact}$ is plotted in Fig.2 for $M_H = 100 \text{ GeV}$ as a function of \sqrt{s} . One can see that it is systematically larger than the total cross-section. This effect is easy to understand if we look at Fig.3, where the contribution of each term in (14) is shown, along with the total cross-section. The main contribution comes from the longitudinally polarized W -bosons, while the contribution of the transversely polarized W -bosons is small. Interestingly, the spin-flip term \mathcal{J}_{LT} , provides a large contribution with a negative sign which cancels a large part of the longitudinal contribution of σ_{LL} into the σ_{total} .

The spin-flip terms ρ_{+0} and ρ_{+-} in (14) depend on $\Delta\tilde{\phi}$ – the relative azimuthal angle defined in the W^+W^- c.m.s. (9), which is different from $\Delta\phi$ (15) in the phase space volume (14) (defined in the e^+e^- c.m.s.).

Would $\Delta\phi$ coincide with $\Delta\tilde{\phi}$, the contribution of interference terms into the total cross section was exactly zero. But these two azimuthal angles coincide only if two c.m. systems moves along their Z axis. In general case $\Delta\tilde{\phi}$ depends on $\Delta\phi$ (see Appendix IV) and the contribution of quantum mechanical interference terms into cross section is non zero and rather large in the region of not very high energy and small Higgs-boson mass ($M_H \approx m_W$) [12,13]. This subtle point was missed in the literature.

From exact formula of Appendix III, taking

$$U = \frac{1}{2}\theta_1\theta_2(1 - \cos \Delta\varphi) + \frac{1}{2}(\theta_1 - \theta_2)^2 \quad (21)$$

we estimate, that e^+e^- lab. system and WW -c.m.s. becomes acollinear when fermion scattering angles $\theta_{1,2}$ are greater then:

$$\theta_m > \frac{4m_w^2}{s \left(1 - \frac{M_H^2}{s}\right)}. \quad (22)$$

The nonvanishing contribution from interference terms \mathcal{J}_{int} in the total cross-section, compared to the longitudinal polarization contribution will be then:

$$\frac{\mathcal{J}_{LT}}{\sigma_{LL}} = -\frac{m_w^2}{M_H^2} \frac{\ln^2 \frac{s\theta_m}{4m_w^2} (1 - M_H^2/s)}{\ln s / M_H^2}, \quad (23)$$

i.e. it vanishes both at small scattering angles, and when $M_H \gg m_w$.

V. Effective vector-boson method

The standard version of the effective W -boson method [1, 2] could be derived from (14) if one takes into account only the terms proportional to $\hat{\sigma}_{LL}$ and $\hat{\sigma}_{TT}$. Since (14) is already written in the factorized form, it is easy to calculate the effective W -boson distribution functions. Of course they coincide with the standard ones. It is important now, to apply these functions only in the kinematical region, where the new additional terms in (14) (which

represents quantum mechanical interference effects) are not important. From Section 4, we have found that it can occur in the case of small scattering angle of electrons (22). It means that there must be a cut off in the integral over dq_i^2 , as q_i^2 is connected with $\cos \theta_i$:

$$q_1^2 = -\frac{s}{2}(1 - \cos \theta_1) \cdot (1 - \zeta) . \quad (24)$$

We have performed computer calculations for each helicity component in the total cross-section σ^{tot} and for σ^{tot} itself for different value of q^2 . The result is presented in Fig.4. We see that the transversal cross section σ_{TT} is small compared to σ^{tot} for any q^2 , and its contribution can be neglected. In the region $q^2 \geq M_H^2 = 100 \text{ GeV}^2$ interference term $-\tau_{LT}(q^2)$ coincides with σ_{LL} and the sum $(\sigma_{LL} + \sigma_{TT})(q^2)$ doesn't correspond to the σ^{tot} for this value of q^2 as it was always considered in the previous calculations. So the natural cut-off for q^2 is $\Lambda^2 = \hat{s}$, where \hat{s} is a characteristic energy of W^+W^- cross-section. The same restriction on q^2 is needed to have W -boson beams to be independent from each other [6]. (For effective photons this cut-off was considered much earlier in [7].)

For $q^2 < \hat{s} = M_H^2$ we have the following approximate formula for σ_{LL} and σ_{TT} :

$$\sigma_{LL}^{appr} = \left(\frac{\alpha}{\sin^2 \theta_w} \right)^3 \frac{1}{16m_w^2} \frac{1}{\left(1 + \frac{m_w^2}{\Lambda^2}\right)^2} \left[\left(1 + \frac{M_H^2}{s}\right) \ln \frac{s}{M_H^2} - 2 \left(1 - \frac{M_H^2}{s}\right) \right] , \quad (25)$$

$$\begin{aligned} \sigma_{TT}^{appr} = & \left(\frac{\alpha}{\sin^2 \theta_w} \right)^3 \frac{1}{16m_w^2} \frac{m_w^4}{M_H} \left[2 \left(1 + \frac{2M_H^2}{s} + \frac{M_H^4}{2s^2}\right) \ln \frac{s}{M_H^2} - 3 \left(1 - \frac{M_H^4}{s^2}\right) \right] \cdot \\ & \cdot \left[\ln \left(\frac{\Lambda^2}{m_w^2} + 1 \right) - \frac{1}{1 + \frac{M_H^2}{\Lambda^2}} \right]^2 . \end{aligned} \quad (26)$$

If we'll absolutely neglect $(-\tau_{LT})$ in this region $q^2 < \Lambda^2$ then we can calculate approximate total cross section. The ratios of $\sigma_{LL}^{appr}/\sigma_{total}^{exact}$ and $(\sigma_{LL} + \sigma_{TT})^{appr}/\sigma_{total}^{exact}$ as functions of \sqrt{s} are plotted in Fig.5(a,b). We have taken cut-off exactly $\Lambda^2 = M_H^2$ and reproduce σ_{total}^{exact} with rather good accuracy. In principle one can also analytically take into account the small contribution of interference term $-\tau_{LT}$ (23). In this case we expect to obtain a little bit smaller cross section just because σ_{tot}^{appr} is obtained from a small part of the phase space, in the same way as we already know from $\gamma\gamma$ -physics [7, 14].

Note, that the same formulas (25-26) are well applied in the case of high energy and heavy Higgs $M_H \gg m_w$ with $\Lambda^2 = M_H^2$, and give the same result as in [2, 3, 10, 11] with a strong suppression of the transversely polarized W -beams, as

$$\frac{\sigma_{TT}}{\sigma_{LL}} \sim \left(\frac{m_w}{M_H} \right)^4 . \quad (27)$$

Conclusions

We have demonstrated that the approximate effective W -boson method can be well applied in the kinematical region where the W^+W^- c.m.s. is collinear with the c.m.s. of the colliding particles, i.e. in the case of small scattering angles for the electrons (or quarks)

and gives the lower bound of the cross-section. It means that there must be introduced a cut-off $\Lambda^2 \lesssim M_H^2$ when $M_H \gg m_w$) or $\Lambda^2 \lesssim m_w^2$ (when $M_H \approx m_w$) in the integrals over the $q^2 - W$ -boson virtualities. In this region the contribution of the transversely polarized beams of W -bosons are always smaller than the leading contribution of longitudinally polarized W -beams.

It was shown, that when these two c.m.s. are not the collinear ones, the negative sign contribution of interference terms plays a crucial role and must be taken into account. That fact was missed in the previous works on this subject.

It is important that the nondiagonal structure of W -boson density matrix is defined only by the fermion- W -fermion vertex and does not depend on the details of WW -interaction. But of course the contribution of each term of the density matrix in the particular subprocess will depend on the dynamics of the WW -interaction and its particular kinematics. So in each particular case all the terms (not only the transversely and longitudinally polarized ones) of the density matrix must be taken into consideration.

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Appendix I

The virtual W -boson density matrix elements:

$$\begin{aligned}
\rho_{00}^{(1)} &= \frac{s^2}{X} \left[\left(\eta - \frac{q_2^2}{s} - \frac{(q_1 q_2)}{s} \right)^2 \frac{X}{s^2} \right] , \\
\rho_{\pm\pm}^{(1)} &= \frac{s^2}{2X} \left[\frac{2X}{s^2} - \left(\eta - \frac{q_2^2}{s} \right) \left(\frac{\hat{s}}{s} - \frac{q_1^2}{s} - \eta \right) + \frac{q_1^2 q_2^2}{s^2} \pm \frac{2\sqrt{X}}{s} \left(\eta - \frac{q_2^2}{s} - \frac{(q_1 q_2)}{s} \right) \right] , \\
\rho_{+-}^{(1)} &= \frac{e^{2i\tilde{\varphi}} s^2}{2X} \left[\left(\frac{\hat{s}}{s} - \frac{q_2^2}{s} - \eta \right) \left(\eta - \frac{q_2^2}{s} \right) - \frac{q_1^2 q_2^2}{s^2} \right] , \\
\rho_{+0}^{(1)} &= -\frac{ie^{i\tilde{\varphi}} s^2}{\sqrt{2}X} \left\{ -\eta + \frac{q_2^2}{s} + \frac{(q_1 q_2)}{s} \pm \frac{\sqrt{X}}{s} \right\} \cdot \left[\frac{q_1^2 q_2^2}{s^2} - \left(\eta - \frac{q_2^2}{s} \right) \left(\frac{\hat{s}}{s} - \frac{q_2^2}{s} - \eta \right) \right]^{1/2} .
\end{aligned}$$

Appendix II

The virtual W -boson density matrix in the case of small scattering angles of the colliding fermions [10]:

$$\begin{aligned}
\rho_{00}^{(1)} &= \frac{4}{\zeta^2} (1 - \zeta) , \\
\rho_{++}^{(1)} &= \frac{2}{\zeta^2} , \\
\rho_{--}^{(1)} &= \frac{2}{\zeta^2} (1 - \zeta) , \\
\rho_{+0}^{(1)} &= -2\sqrt{2}ie^{i\tilde{\varphi}_1} \frac{1}{\zeta^2} (1 - \zeta)^{1/2} , \\
\rho_{0-}^{(1)} &= -2\sqrt{2}ie^{i\tilde{\varphi}_1} \frac{1}{\zeta^2} (1 - \zeta)^{3/2} , \\
\rho_{+-}^{(1)} &= -2e^{2i\tilde{\varphi}_1} \frac{1}{\zeta^2} (1 - \zeta) ,
\end{aligned}$$

where $\zeta = \frac{2\omega_1}{\sqrt{s}}$. The analogous relation for $\rho_{ab}^{(2)}$ can be found with a substitution $\frac{2\omega_1}{\sqrt{s}} \rightarrow \frac{2\omega_2}{\sqrt{s}} = \eta$; $\varphi_1 \rightarrow -\varphi_2$ (and $q_2^2 \rightarrow q_1^2$ in Appendix I).

Appendix III

For the exact numerical calculation of the total cross-section (10-13), we used the following final formula:

$$\frac{d\sigma \cdot 2\pi}{d\Delta\varphi} = \frac{\alpha_w^3 m_w^2}{4s^2} \int_{-1}^1 d\cos\theta_1 \int_{-1}^1 d\cos\theta_2 \int_{M_H^2/s}^1 d\zeta \cdot$$

$$\cdot \frac{1}{\left(1 + \frac{2m_w^2}{s(1-\eta_0)} - \cos\theta_2\right)^2 \cdot \left(1 + \frac{2m_w^2}{s(1-\zeta)} - \cos\theta_1\right)^2} \cdot \frac{(1-U)}{U + (1-U)\zeta} ;$$

where $\eta_0 = \frac{m_H^2/s + (1-\zeta)U}{\zeta(1-U)+U}$; $U = \frac{1}{2} (1 - \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\Delta\varphi)$

$$(0 \leq U \leq 1).$$

Appendix IV

For the exact numerical calculations of the helicity components in the total cross-section (14) (with (9), (15)-(20)) we used as the final formula:

$$\frac{2\pi d\sigma}{d\Delta\varphi} = \frac{m_w^2 \alpha_w^3}{2^6 s^2} \int_{-1}^1 d\cos\theta_1 \int_{-1}^1 d\cos\theta_2 \int_{\frac{M_H^2}{s}}^1 d\zeta \cdot$$

$$\cdot \left\{ \rho_{00}^{(1)} \rho_{00}^{(2)} \frac{(q_1 q_2)^2}{q_1^2 q_2^2} + \rho_{++}^{(1)} \rho_{++}^{(2)} + \rho_{--}^{(1)} \rho_{--}^{(2)} + 2 \cos 2\Delta\tilde{\varphi} |\rho_{+-}^{(1)}| |\rho_{+-}^{(2)}| - \right.$$

$$\left. - \frac{2(q_1 q_2)}{\sqrt{q_1^2 q_2^2}} \cos \Delta\tilde{\varphi} (|\rho_{+0}^{(1)}| |\rho_{+0}^{(2)}| + |\rho_{0-}^{(1)}| |\rho_{0-}^{(2)}|) \right\} ,$$

where η_0 , U are the same as in the Appendix III;

$$q_2^2 = -\frac{s}{2}(1-\eta_0)(1-\cos\theta_2); \quad q_1^2 = -\frac{s}{2}(1-\zeta)(1-\cos\theta_1); \quad (q_1 q_2) = \frac{M_H^2 - q_1^2 - q_2^2}{2};$$

$$\cos \Delta\tilde{\varphi} = \frac{s}{2\sqrt{q_1^2 q_2^2}} .$$

$$\frac{\eta_0 \zeta \left(\frac{M_H^2}{s} - \frac{q_1^2}{s} - \frac{q_2^2}{s} \right) - \zeta \frac{q_2^2}{s} \left(\frac{M_H^2}{s} + \frac{q_1^2}{s} - \frac{q_2^2}{s} \right) - \eta_0 \frac{q_1^2}{s} \left(\frac{M_H^2}{s} + \frac{q_2^2}{s} - \frac{q_1^2}{s} \right) - \frac{4X}{s^2} + \frac{2M_H^2 q_1^2 q_2^2}{s^3}}{\left[\left(\zeta - \frac{q_1^2}{s} \right) \left(\zeta - \frac{M_H^2}{s} \right) + \zeta \cdot \frac{q_2^2}{s} \right]^{1/2} \left[\left(\eta_0 - \frac{q_2^2}{s} \right) \left(\eta_0 - \frac{M_H^2}{s} \right) + \eta_0 \frac{q_1^2}{s} \right]^{1/2}}$$

$$\cos 2\Delta\tilde{\varphi} = 2 \cos^2 \Delta\tilde{\varphi} - 1 .$$

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FIGURE CAPTIONS

- Fig. 1 Feynman graph for Higgs-boson production via W^+W^- fusion in e^+e^- (or $q\bar{q}$) collisions.
- Fig. 2 The exact numerical calculation of the contribution of only diagonal terms of W -density matrix into the total cross-section. It is seen that without interference terms it overestimates almost 3 times the total cross-section.
- Fig. 3 The exact numerical calculation of the contribution of each polarization of the W -amplitude in the total cross-section. It is very well seen the cancelation of σ_{LL} by \mathcal{J}_{LT} - the interference term.
- Fig. 4 The exact computer calculation of the dependence of each polarization of W -amplitude on the W -boson virtuality q^2 . One can see that at $q^2 = \Lambda^2 = M_H^2$ there is the total cancelation between longitudinal and interference terms. It means that the integration over dq^2 in the approximate calculations via effective W -boson method has a cut-off $\Lambda \sim M_H$.
- Fig. 5 The ratio of the approximate effective W -boson method calculation of σ_{LL}^{appr} (a) and $(\sigma_{LL}^{ww} + \sigma_{TT}^{ww})^{appr}$ (b) with $q^2 < \Lambda^2 = M_H^2$ over the exact computer calculation σ_{exact} . The ratio is always less than 1 above the threshold, it means that the effective W -boson method gives the lower bound of the cross section.